

A Discrete Backlash Operator for Identification and Compensation of Hysteresis and Non-linearity for Piezoelectric Actuator

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Abstract— A new invertible digitized model called Discrete Backlash Operator is proposed to model complex hysteretic nonlinearities. A discrete backlash operator is formed by combining multiple elementary virtual gears called discrete backlash gears. A discrete backlash operator with n discrete backlash gears has 2^n possible states. The inverse model of discrete backlash operator is also developed for a feedforward controller to control the piezoelectric actuator.

I. INTRODUCTION

Piezoelectric actuators are popular in real-time micromanipulation applications due to their rapid and accurate response. Handheld microsurgical devices such as Micron [1] and iTrem [2] uses piezoelectric actuators for their micromanipulators. The effective control of piezoelectric actuators is limited by their undesired complex hysteretic nonlinearities.

To overcome the limitation due to complex hysteretic nonlinearities and to achieve the micro-meter scale accuracy, micromanipulation systems typically deploy various types of controllers. The controller of an electric charge control system uses induced charge to control a piezoelectric actuator instead of applying voltage to it [3], [4] because the relationship between deformation of a piezoceramic and the induced charge has less hysteresis. Requirement of specialized equipments to measure and amplify the induced charge makes this approach unsuitable for small, space constrained systems such as our handheld instrument.

Another approach uses closed-loop control systems to reduce the error caused by nonlinear hysteresis of piezoelectric actuators. This approach also is not a suitable choice to control micromanipulator for real-time high frequency hand tremor compensation because of requirement for accurate displacement sensor and intrinsic stability problem [5].

Feedforward controllers improve the accuracy of hysteretic actuator response by implementing inverse function of a mathematical hysteresis model that closely represents the complex hysteresis behaviour of the actuator. Due to its simplicity and analytically computable inverse, Prandtl-Ishlinskii model [6] is well known among several mathematical models that have been proposed to describe the complex hysteretic nonlinearities.

Several derivative models based on Prandtl-Ishlinskii model have been proposed. To account for the hysteretic nonlinearity at varying actuation frequency, Ang et al. [7] proposed a rate-dependent modified Prandtl-Ishlinskii operator. Tan et al. [8] proposed an extended Prandtl-Ishlinskii operator to tackle ill-conditioned situations. Jiang et al. [9] proposed a modified Prandtl-Ishlinskii operator to model the asymmetric hysteresis nonlinearity of piezoelectric actuators.

Unlike existing approaches, a new digitized model for hysteretic nonlinearity is proposed in this paper. The proposed model and its inverse are simpler than the Prandtl-Ishlinskii model while maintaining the comparable modeling accuracy. The amount of computation and calculation time are also lesser.

II. Modeling of Hysteresis

In this paper, a new model called Discrete Backlash Operator is proposed to model complex hysteretic nonlinearities. A discrete backlash operator is formed by combining multiple elementary virtual gears called discrete backlash gears. An elementary discrete backlash gear has two properties, ΔX and ΔY , to define its input and output characteristics. It also has a bistable state, b , which can be either 0 or 1. A discrete backlash operator with n discrete backlash gears has 2^n possible states. The current state of discrete backlash operator is represented by a binary number (B) as shown in (1).

$$B = b_{n-1} \dots b_1 b_0 \quad (1)$$

where b_0 is the state of the least significant gear and b_{n-1} is that of the most significant gear. A discrete backlash operator has the following variables that depend on its current state.

- X The input value that corresponds to the current state. The value of X can be represented by (2).
- Y The output value that corresponds to the current state. The value of Y can be represented by (3).
- S The number that identifies the index of the least significant one bit of the current state.
- Z The number that identifies the index of the least significant zero bit of the current state.

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$$X_B = X_0 + \sum_{i=0}^{n-1} b_i \times \Delta X_i \quad (2)$$

$$Y_B = Y_0 + \sum_{i=0}^{n-1} b_i \times \Delta Y_i \quad (3)$$

where X_0 and Y_0 are the X and Y coordinates of the operator when its state, B , is zero.

The value of S is found by using a bitwise operation called find first set. Similarly, the bitwise operation find first zero is used to find the value of Z . They are common operations in digital computing. Many computer architectures include instructions implemented in hardware to rapidly perform these operations. If these operations are not available in hardware, they can be efficiently implemented in software by using an algorithm such as de Bruijn Sequences [10].

S and Z are obtained from the current state B as follows.

$$S = f^1(B) \quad (4)$$

$$Z = f^0(B) \quad (5)$$

where f^1 is find first set operation and f^0 is find first zero operation. The returned values of $f^1(0)$ and $f^0(2^n - 1)$ for a discrete backlash operator are defined as n .

A state of a discrete backlash operator can change to either one of its two neighbours, upper state or lower state. When a current state B goes up to next upper state, its bit b_Z is set. The relations between the properties of the current state, B_k , and the next state, B_{k+1} can be represented recursively as follows.

$$X_{B_{k+1}} = X_{B_k} + \Delta X_Z \quad (6)$$

$$Y_{B_{k+1}} = Y_{B_k} + \Delta Y_Z \quad (7)$$

Similarly, when it goes down to next lower state, its bit b_S is cleared. The relations between the properties of the next state and the current state are as follows.

$$X_{B_{k+1}} = X_{B_k} - \Delta X_S \quad (8)$$

$$Y_{B_{k+1}} = Y_{B_k} - \Delta Y_S \quad (9)$$

For the piecewise linear representation of a hysteresis curve shown in Fig. 1, there are two linear segments as the curve goes up from the bottom to the top. Therefore, it can be represented by two discrete backlash gears. The first segment corresponds to the least significant gear and the last segment corresponds to the most significant gear.

In general, a discrete backlash operator with n discrete backlash gears is represented by

$$\mathbf{H} = \begin{bmatrix} \Delta X_0 & \dots & \Delta X_{n-1} \\ \Delta Y_0 & \dots & \Delta Y_{n-1} \end{bmatrix} \quad (10)$$

where the n columns of the matrix represent n discrete backlash gears. A discrete backlash operator with three gears has eight possible states as shown in Fig. 2.

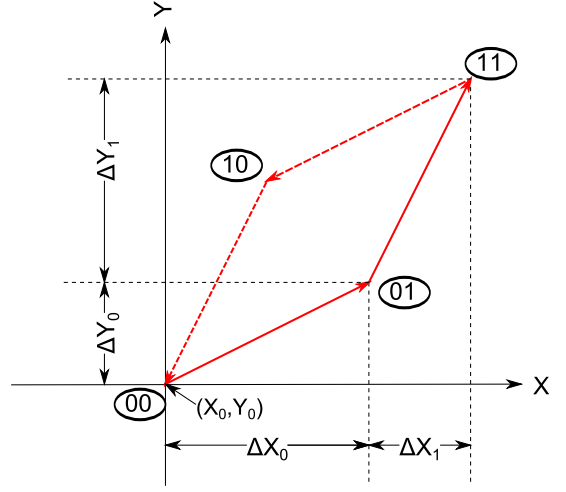


Fig. 1. Hysteresis curve for discrete backlash operator with two gears.

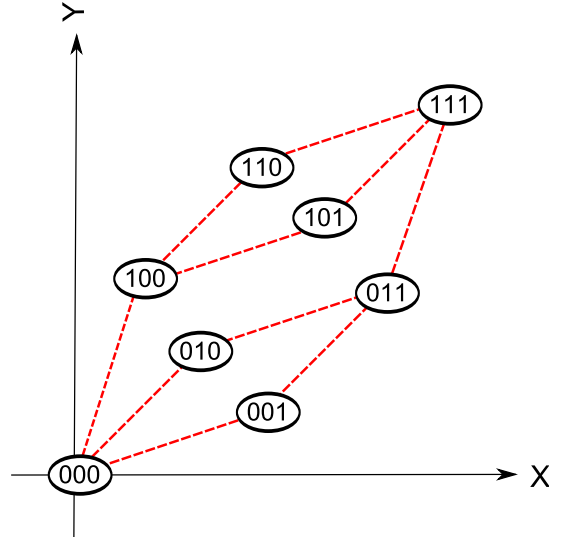


Fig. 2. Hysteresis curve for discrete backlash operator with three gears.

Each time a discrete backlash operator with current state B_k receives an input, it performs the following procedure to produce the corresponding output.

1. If the input value, x , is greater than or equal to X_{B_k} and less than $X_{B_k} + \Delta X_Z$, the value of the output y is calculated using (11).
2. If the input value, x , is greater than or equal to $X_{B_k} + \Delta X_Z$, change the state of the operator to its upper neighbour by setting b_Z , update the value of X_{B_k} and Y_{B_k} using (6) and (7), and repeat the procedure.
3. If the input value, x , is less than X_{B_k} and greater than $X_{B_k} - \Delta X_S$, the value of the output y is calculated using (12).
4. If the input value, x , is less than or equal to $X_{B_k} - \Delta X_S$, change the state of the operator to its lower neighbour by clearing b_S , update

the value of X_{B_k} and Y_{B_k} using (8) and (9), and repeat the procedure.

$$y = Y_{B_k} + \frac{\Delta Y_Z}{\Delta X_Z} \times (x - X_{B_k}) \quad (11)$$

$$y = Y_{B_k} + \frac{\Delta Y_S}{\Delta X_S} \times (x - X_{B_k}) \quad (12)$$

III. Discrete Point Operator

The output of the discrete backlash operator has odd symmetry property to the centre point of the hysteresis loop. In fact, most of the hysteresis loops produced by real actuators are not symmetric. Therefore, another operator that represents asymmetrical, memory free nonlinearities is usually combined with such a symmetrical hysteresis operator to overcome the problem. In this section, another modeling technique called discrete point operator is proposed which is similar to the discrete backlash operator to model memory free nonlinearities.

A discrete point operator is composed of a certain number of points each with two properties ΔX and ΔY to represent the input and output relationship. The current state of a discrete point operator with order m is represented by a number, C . The range of C is $0 \leq C \leq (m-1)$. A discrete point operator with three linear segments is shown in Fig. 3.

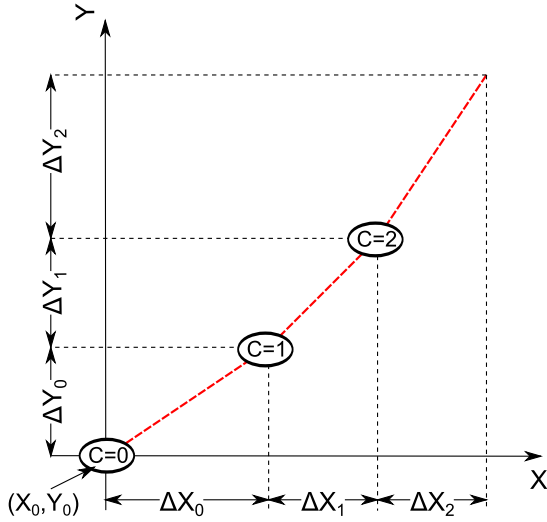


Fig. 3. Nonlinear curve for discrete point operator with three points.

The input and output values corresponding to the current state, C , of a discrete point operator is as follows.

$$X_C = X_0 + \sum_{i=0}^{C-1} \Delta X_i; \quad C = 1, \dots, m-1 \quad (13)$$

$$Y_C = Y_0 + \sum_{i=0}^{C-1} \Delta Y_i; \quad C = 1, \dots, m-1 \quad (14)$$

where X_0 and Y_0 are the X and Y coordinates of the operator when its state, C , is zero.

In general, a discrete point operator with m segments is represented by

$$\mathbf{S} = \begin{bmatrix} \Delta X_0 & \dots & \Delta X_{m-1} \\ \Delta Y_0 & \dots & \Delta Y_{m-1} \end{bmatrix} \quad (15)$$

where each column represents each point of the operator. Each time a discrete point operator with current state C receives an input, it performs the following procedure to produce the corresponding output.

1. If the input value, x , is greater than or equal to X_C and less than or equal to $X_C + \Delta X_C$, the value of the output y is calculated using (16).
2. If the input value, x , is greater than $X_C + \Delta X_C$, change the state of the operator to its upper neighbour by increasing C if it is less than $m-1$, update the value of X_C and Y_C by adding ΔX_C and ΔY_C respectively, and repeat the procedure.
3. If the input value, x , is less than X_C and C is greater than 0, change the state of the operator to its lower neighbour by decreasing C , update the value of X_C and Y_C by subtracting ΔX_C and ΔY_C respectively, and repeat the procedure.

$$y = Y_C + \frac{\Delta Y_C}{\Delta X_C} \times (x - X_C) \quad (16)$$

IV. Inverse Model

The inverse model of the discrete backlash operator produces the hysteresis path which is reflection of the original path along the 45° line. A linear response is obtained by cascading the inverse hysteresis operator Γ^{-1} as a feedforward controller with the actual hysteresis, which is represented by the hysteresis operator Γ . The proposed model for the inverse feedforward controller is illustrated in Fig. 4.

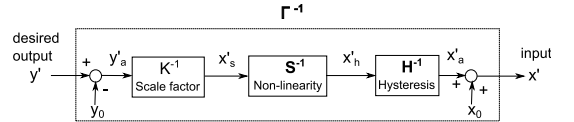


Fig. 4. Block diagram of the inverse discrete backlash operator as a feedforward controller.

The parameters of the inverse discrete backlash operator and the inverse discrete point operator can be found by simply exchanging ΔX with ΔY for each of their basic element.

$$\Delta X'_i = \Delta Y_i; \quad i = 0, \dots, n-1 \quad (17)$$

$$\Delta Y'_i = \Delta X_i; \quad i = 0, \dots, n-1 \quad (18)$$

Due to its recursive nature, the amount of calculation for discrete backlash operator does not depend on the

number of segments, n , while that of PI operator is directly proportional to the order, n . As a result, the amount of calculation required for a feedforward controller using discrete backlash operator and discrete point operator is much lesser than that of PI and dead-zone operator.

V. Experimental Results

In this section, the hysteretic nonlinearity of a multi-layered piezoceramic, PICMA P-885.91 (Physik Instrumente, Karlsruhe, Germany), is modeled using discrete backlash operator. Two experiments have been performed. The first experiment is to observe the behaviour of the piezoelectric actuator and to obtain the parameters of the model. The second experiment is to test the performance of the feedforward controller deploying the inverse discrete backlash operator.

At first, the responses of the piezoelectric actuator to periodic control inputs is measured experimentally to find the hysteresis model parameters. The control input applied to the piezoelectric actuator to model its behaviour is 1 Hz sinusoidal with a full-scale 3.6 V peak to peak as shown in Fig. 5. The measured piezoelectric actuator response to that input is illustrated in Fig. 6 with red line and in Fig. 7 with green dotted line. Least-squares fitted discrete backlash operator model ($n=25$) is superimposed on the measured response with blue solid line. The values of K , X_0 , and Y_0 are 12.53, 0, and -22.97 respectively.

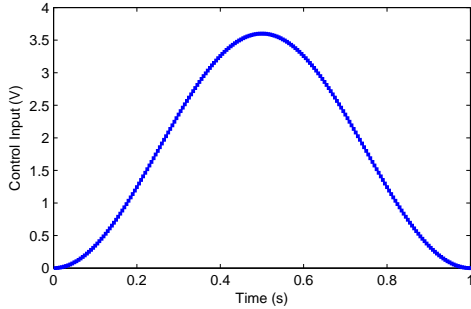


Fig. 5. Control Input which is applied to the piezoelectric actuator to model the behaviour of its hysteretic nonlinearity.

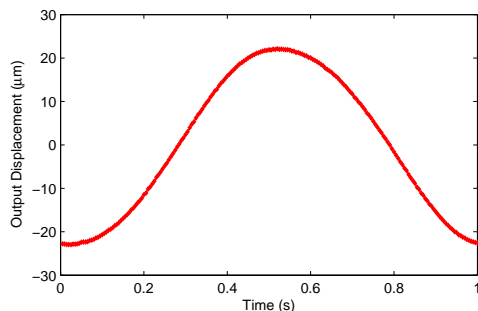


Fig. 6. The response of the piezoelectric actuator versus time.

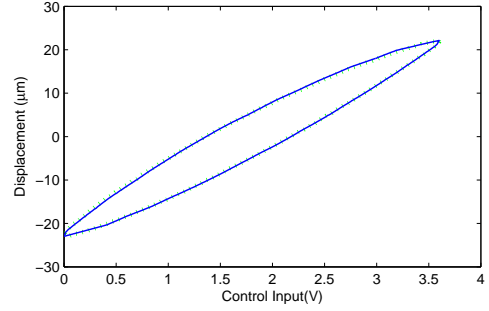


Fig. 7. Light green dashed lines are the measured piezoelectric actuator response and blue solid lines are the identified discrete backlash operator ($n=25$) model.

To test the performance of the feedforward controller deploying the inverse model, a 1 Hz sinusoidal signal with gradually increasing amplitude from 0 μm to 20 μm is used as the control input and the response of the piezoelectric actuator is measured. The RMSE between the desired control input and the measured actuator response without compensation is 1.68 μm (Fig. 8, Fig. 9, and Fig. 10).

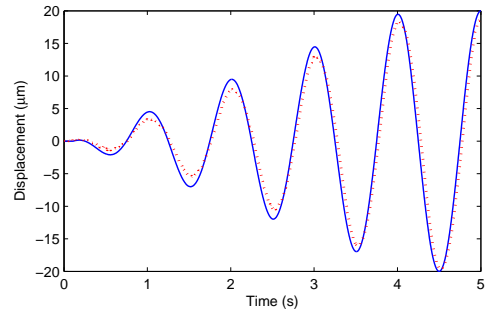


Fig. 8. Solid blue lines are desired control input and red dotted lines are the response of the piezoelectric actuator when inverse feedforward controller is not deployed.

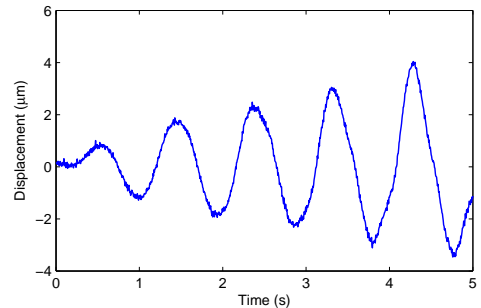


Fig. 9. Error between the desired control input and the actual actuator response when inverse feedforward controller is not deployed.

Thereafter, a feedforward controller deploying the inverse model is used to drive the piezoelectric actuator to obtain an identity mapping between the desired actuator output and actual actuator response. The inverse

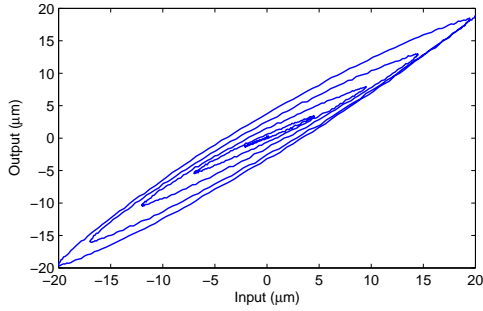


Fig. 10. Input and output displacement relationship when inverse feedforward controller is not deployed.

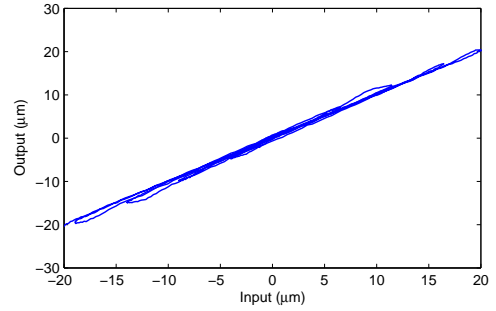


Fig. 13. Input and output displacement relationship when inverse feedforward controller is deployed.

operator is obtained by using the method mentioned in the previous section. The RMSE between the desired control input and the measured actuator response with compensation is reduced to $0.58 \mu\text{m}$ (Fig. 11, Fig. 12, and Fig. 13).

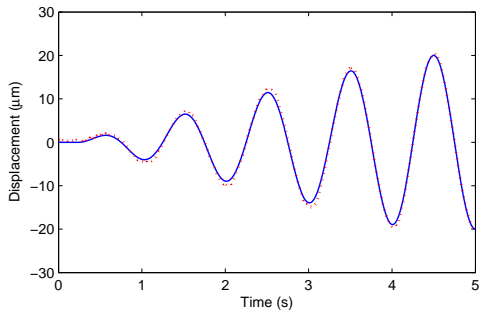


Fig. 11. Solid blue lines are desired control input and red dotted lines are the response of the piezoelectric actuator when inverse feedforward controller is deployed.

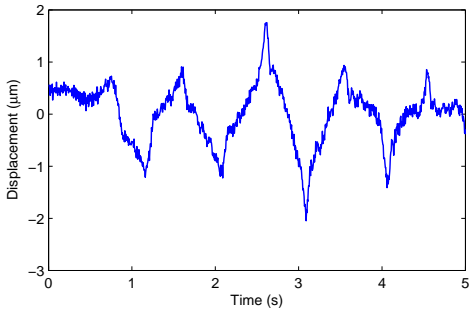


Fig. 12. Error between desired control input and the actual actuator response when inverse feedforward controller is deployed.

VI. Conclusion

The discrete backlash operator is proposed as an alternative to Prandtl-Ishlinskii modeling approach. The advantages of the discrete backlash operator are its simpler and intuitive model, less amount of calculation, and simpler parameter identification. The proposed model can be easily extended or modified to handle its

response behaviour for out of range input. The amount of calculation is significantly reduced for a large value of n . For a smaller n , the output values for their corresponding states can be stored in a lookup table and the amount of calculation can be reduced further. Due to its simple nature, the discrete backlash operator is suitable for applications that requires very high sampling rate or for the systems using low-end digital hardware.

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